Quantencomputer – Steuerbare Entanglement-Quelle in elastischer Elektron-Austausch-Streuung von chaotischen Spin-Systemen

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Contents

Introduction

Introduction

Most intriguing phenomenon in nature.
Continuously at the center of intense research activities,
Brunner *et al* (2014); Chandra and Ghosh (2013);
Gühne and Tóth (2009); Horodecki *et al* (2009); Reid *et al* (2009).

Entanglement

$\downarrow$

Fundamental to quantum mechanics

non-locality, superposition principle, *Verschränkung* by Schrödinger (1935).

$\downarrow$

Local realism vs. Quantum mechanics

Bell's theorem (Bell, 1964)
Hensen *et al* (2015); Giustina *et al* (2015); Shalm *et al* (2015).

$\downarrow$

AND

$\swarrow \quad \searrow$

Moreover

Entanglement

↙ ↓ ↘

QI & QC – Viable resource for:

- Quantum cryptography
  Herbert (1975); Ekert (1991),
- Quantum teleportation
  Bennett *et al* (1993),
- Quantum computation
  Gottesmann and Chuang (1999);
  Raussendorf and Briegel (2001).

New field in scattering physics:

- Scattering process yields tunable entanglement resource.
- Proof of intrinsic non-locality in exchange processes.
- Relating scattering physics and Bell's theorem.
- Maximal violation of Bell's inequalities.

↘ ↓ ↙

☞ **This talk and our results shall interrelate all these areas.** ☜

K. Blum and B. Lohmann, *Phys. Rev. Lett.* **116**, 033201 (2016),
B. Lohmann, K. Blum, and B. Langer, *Phys. Rev. A* **94**, 032331 (2016).

Scattering Theory

Scattering Theory

- Consider the **incoming state** of the particles as the state vector $|\psi_{in}\rangle$ at an **infinitely remote past** when the **interaction** between the particles **can be neglected**.

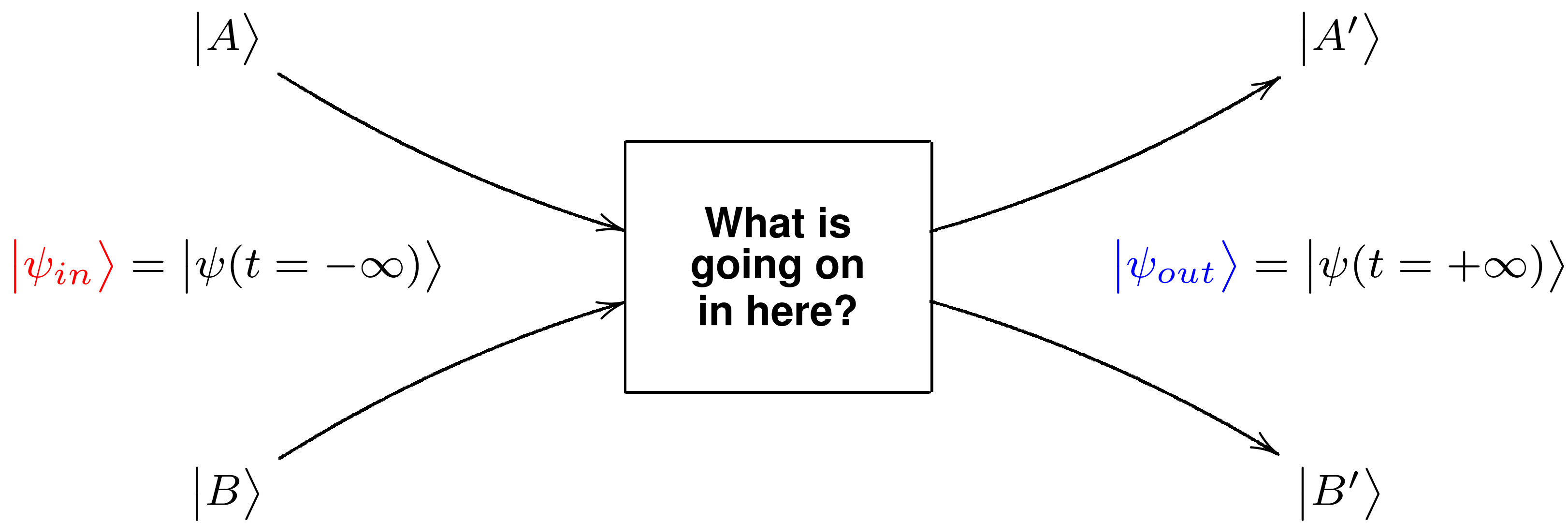

FIG. 1 A typical *black box* scattering scenario.

- The **outgoing state** $|\psi_{out}\rangle$ describes the state in a **large distance** from the collision at an **infinitely late future** instant.
- The distance between the particles is assumed as large enough for the **interaction to be neglected again**.

Scattering Operator

- The collision is thought of as a **black box**, mathematically represented by the **unitary operator** S, which transforms the **in** states into the **out** states.

☞ The S **matrix** can then be defined by the relation

$$|\psi_{out}\rangle = S\,|\psi_{in}\rangle\ . \tag{1}$$

- When the **initial state** is represented by the **density matrix**

$$\rho_{in} = \sum_i p_i |\psi_{in}^{(i)}\rangle\langle\psi_{in}^{(i)}|\ , \tag{2}$$

with probabilities $p_i \geq 0$.

- The density matrix ρ'_{out} describing the **final particles** is obtained by sandwiching ρ_{in} between S and S^+,

$$\rho'_{out} \equiv S\rho_{in}S^+ = \sum_i p_i S|\psi_{in}^{(i)}\rangle\langle\psi_{in}^{(i)}|S^+ = \sum_i p_i|\psi_{out}^{(i)}\rangle\langle\psi_{out}^{(i)}|\ . \tag{3}$$

Transition Operator

- Since we are only interested in **transitions between different states** it is convenient to extract from S the identity operator $\mathbb{1}$ and define the **transition operator** T as

$$T = S - \mathbb{1} \; . \tag{4}$$

- It follows

$$T\left|\psi_{in}\right\rangle = \left|\psi_{out}\right\rangle - \left|\psi_{in}\right\rangle \; . \tag{5}$$

- All possible transitions (scattering, reactions) in the collision system will be connected with the **dissimilarity between initial and final state.**

- **The T operator transforms the in state into the scattered out state.**

- Thus, the **interesting part** of the density matrix ρ'_{out} is that one which contains the information on the **scattered states alone**, which is given by

$$\rho_{out} = T \rho_{in} T^{+} \; . \tag{6}$$

- The **central problem of scattering theory** is the determination of T, that is to **determine all elements of the T matrix.**

Density Matrix of the Scattering System

Elastic Scattering

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- Initially unpolarized electrons and unpolarized atoms:

$$e + A(H, Li, Na)\,{}^2\mathrm{S}_{1/2} \longrightarrow e + A\,{}^2\mathrm{S}_{1/2}\ . \tag{7}$$

- Quasi **hydrogen-like** (one electron) atoms; *H, Li, Na*.
  - **Light atoms** $\longrightarrow$ spin-orbit interaction can be neglected.
  - **Coulomb- and exchange forces**, only.

☞ **Initial unpolarized state** density matrix:

$$\rho_{in} = \frac{1}{4} \sum_{m_1 m_2} |m_1 m_2\rangle\langle m_1 m_2|\ . \tag{8}$$

Spin Density Matrix

- Calculate **final state density matrix** after scattering:

$$\rho = T \rho_{in} T^{+} \ , \tag{9}$$

where T is the transition operator.

☞ Clebsch-Gordan Algebra yields:

$$\rho = \frac{1}{8\sigma} \begin{pmatrix} 2|f^{(1)}|^2 & 0 & 0 & 0 \\ 0 & |f^{(1)}|^2 + |f^{(0)}|^2 & |f^{(1)}|^2 - |f^{(0)}|^2 & 0 \\ 0 & |f^{(1)}|^2 - |f^{(0)}|^2 & |f^{(1)}|^2 + |f^{(0)}|^2 & 0 \\ 0 & 0 & 0 & 2|f^{(1)}|^2 \end{pmatrix} , \tag{10}$$

$f^{(S)} = \langle SM_s | T | SM_s \rangle$, denotes the **triplet** $(S = 1)$ and **singlet** $(S = 0)$ **scattering amplitudes** (T matrix is independent of M_s).

- Here, ρ is **normalized** by the **differential cross section**:

$$\sigma = \frac{1}{4}\left(3|f^{(1)}|^2 + |f^{(0)}|^2\right) , \qquad \text{and} \qquad \mathrm{tr}\ \rho = 1 \ . \tag{11}$$

Spin correlations

- **Characterization** of the 4×4 spin density matrix (10):

$$\rho = \frac{1}{4}\left[\mathbb{1}\times\mathbb{1}+\sum_i P_i^{(1)}(\sigma_i\times\mathbb{1})+\sum_j P_j^{(2)}(\mathbb{1}\times\sigma_j)+\sum_{ij}\left(P_i^{(1)}\times P_j^{(2)}\right)(\sigma_i\times\sigma_j)\right] . \tag{12}$$

  - **Two** individually measured **spin polarization** vectors $\mathbf{P}^{(1)}$ and $\mathbf{P}^{(2)}$, referring to particles 1 and 2:

$$P_i^{(1)} = \mathrm{tr}\,\rho\big(\sigma_i \times \mathbb{1}\big)\ , \quad \text{and} \quad P_i^{(2)} = \mathrm{tr}\,\rho\big(\mathbb{1} \times \sigma_i\big)\ , \tag{13}$$

  ($i = x, y, z$), 2-dim. unit matrix $\mathbb{1}$, and $\times$ denotes the direct product.

  - **Nine** direct product components $P_i^{(1)} \times P_j^{(2)}$ of the **spin–spin correlation tensor** $(i, j = x, y, z)$:

$$P_i^{(1)} \times P_j^{(2)} = \mathrm{tr}\,\rho\big(\sigma_i \times \sigma_j\big)\ . \tag{14}$$

- σ_i $(i = x, y, z)$ abbreviates the Pauli matrices.

- The **simple structure** of the density matrix (10) allows for quickly calculating the **relevant parameters**.
- The individual polarization vectors of the two sub-systems cancel,

$$\mathbf{P}^{(1)} = 0 \qquad \text{and} \qquad \mathbf{P}^{(2)} = 0 \ . \tag{15}$$

- All spin-spin correlation tensors with $i \neq j$ vanish.

☞ **The only non-vanishing** spin correlation parameters are

$$P = P_i^{(1)} \times P_i^{(2)} = \frac{|f^{(1)}|^2 - |f^{(0)}|^2}{3|f^{(1)}|^2 + |f^{(0)}|^2} \ , \quad i = x, y, z \ , \tag{16}$$

- Introducing the **parameter** $\mathbf{P} = \mathbf{P}(\boldsymbol{\theta}, \mathbf{E})$ which is a function of scattering angle and energy.

✓ In general, $\mathbf{P}$ **is restricted** to the interval $[-1, 1]$. From(16) we get the further restriction

$$-1 \leq P \leq \frac{1}{3} \ . \tag{17}$$

$\rho = \rho(P)$

- The **spin density matrix** (10) can be expressed in terms of the spin correlation parameter. We obtain

$$\rho = \frac{1}{4}\begin{pmatrix} 1+P & 0 & 0 & 0 \\ 0 & 1-P & 2P & 0 \\ 0 & 2P & 1-P & 0 \\ 0 & 0 & 0 & 1+P \end{pmatrix} . \tag{18}$$

☞ The **key equation** (18) shows that the spin system is completely characterized by the **single parameter P**.

- For **negative values** of $P = -|P|$, that is **anticorrelated spins**, we can write the scattering matrix ρ in form of a **Werner state** (Werner, 1989)

$$\rho = \frac{1-|P|}{4}\mathbb{1} + |P|\big|00\big\rangle\big\langle 00\big| \, , \tag{19}$$

with the **singlet state** $\quad \big|00\big\rangle = \frac{1}{\sqrt{2}}\big(\big|\uparrow\downarrow\big\rangle - \big|\downarrow\uparrow\big\rangle\big) \; . \qquad (20)$

Coincidence Measurements of Spin-Spin Correlation Parameter

Coincidence measurements

- The **correlation parameters** refer to experiments where both scattered particles are measured in **coincidence by two observers**.

- E.g., $P_z^{(1)} \times P_z^{(2)}$ gives the outcome of an experiment where **both** analyzer-detector sets are oriented **along the $\mathbf{z}$-direction**, which yields (Blum, 2012)

$$P_z^{(1)} \times P_z^{(2)} = \frac{1}{N} \left(N(z)_{\uparrow\uparrow} + N(z)_{\downarrow\downarrow} - N(z)_{\uparrow\downarrow} - N(z)_{\downarrow\uparrow} \right) , \tag{21}$$

  where, e.g. $N(z)_{\uparrow\uparrow}$ denotes the number of measurements finding both particles with spin up with respect to the z-axis.

- $\longrightarrow$ **This is a difficult experiment.**

■ **Measuring** the spin-spin correlations along **directions** $\mathbf{a}$ and $\mathbf{b}$ is given by

$$☞ \quad P_a^{(1)} \times P_b^{(2)} = P(\theta, E) \cos\beta \ . \quad ☜ \tag{22}$$

■ This and (16) exhibit the **rotational symmetry** of the spin–spin system.

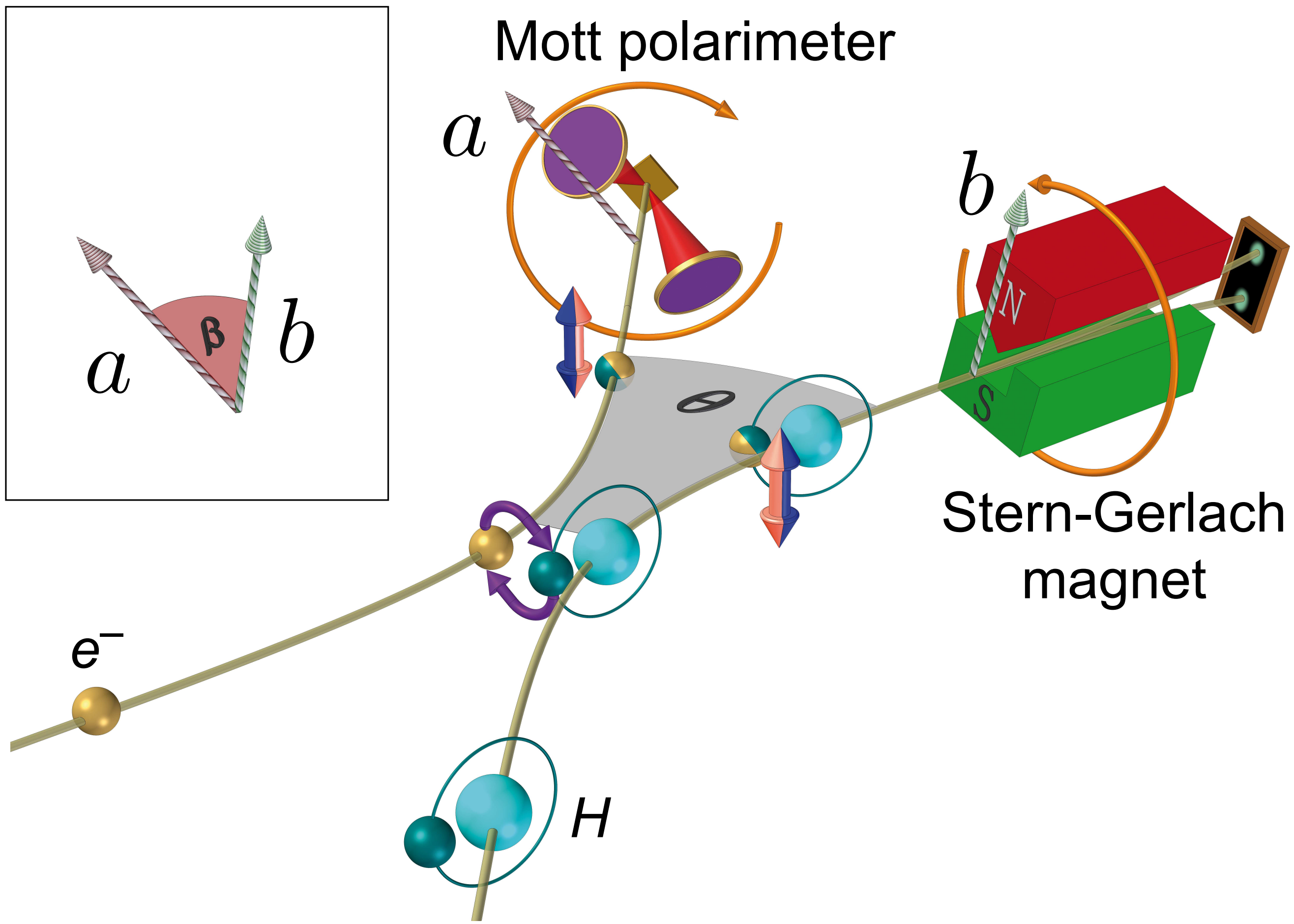

FIG. 2 Scheme of an e–H spin correlation coincidence experiment.

Spin asymmetry A_{ex}

- A spin selective coincidence experiment as described above is not easy to perform.

☞ **Important:** the correlation parameter (16) can also be obtained from the results of a **different type of experiment**.

👍 **Many experimental and numerical results** are already available.

- Consider the case where electrons and atoms, **both**, are **initially polarized**.

$$e\uparrow + A\uparrow \longrightarrow e + A \qquad \Longrightarrow \sigma_{\uparrow\uparrow}\,, \tag{23}$$

$$e\uparrow + A\downarrow \longrightarrow e + A \qquad \Longrightarrow \sigma_{\uparrow\downarrow}\,, \tag{24}$$

where $\sigma_{\uparrow\downarrow}$ and $\sigma_{\uparrow\uparrow}$ denote differential cross sections for incident parallel and antiparallel spins, respectively.

☞ In this case, the **spin asymmetry** A_{ex} can be measured

$$A_{ex} = \frac{\sigma_{\uparrow\downarrow} - \sigma_{\uparrow\uparrow}}{\sigma_{\uparrow\downarrow} + \sigma_{\uparrow\uparrow}}\,, \tag{25}$$

A_{ex} vs P

- Expressing $\boldsymbol{A_{ex}}$ in terms of the **scattering amplitudes** $f^{(0)}$ and $f^{(1)}$, we obtain (Lorentz *et al*, 1991)

$$A_{ex} = \frac{\left|f^{(0)}\right|^2 - \left|f^{(1)}\right|^2}{3\left|f^{(1)}\right|^2 + \left|f^{(0)}\right|^2} \,. \tag{26}$$

A comparison with (16) yields the **simple relation**

$$P(\theta, E) = -A_{ex}(\theta, E) \,. \tag{27}$$

- Using published data for the spin asymmetry A_{ex}, then (27) allows to immediately obtain the spin correlation parameter P for our considered **experimental setup with initially unpolarized particles**.

See the discussion of data later on.

Entanglement versus Separability

Entanglement vs Separability

- Under **which conditions** is the density matrix ρ, i.e. the mixed spin state (18), separable or entangled, or a combination of both?
- Generally, a density matrix of a bi-partite mixed state is **separable if and only if**

$$\rho = \sum_{i=1}^{n} p_i \left|a_i\right\rangle\left\langle a_i\right| \times \left|b_i\right\rangle\left\langle b_i\right| \,, \qquad p_i \geq 0 \,, \tag{28}$$

  where the **pure one-particle states** $\left|a_i\right\rangle$ and $\left|b_i\right\rangle$ refer to the first (electron) and second (atom) particle, respectively.
- If **no transformation** of a given density matrix ρ to the form (28) can be given, the system is said to be **non-separable or entangled**.

- Generally, this can be a **hard task**.

- **Convenient criterion** by Peres (1996) and Horodecki *et al* (1996).

☞ In our case of a 4×4 density matrix, yields a **necessary and sufficient condition** for separability.

Peres-Horodecki Criterion

- For this, we construct the **partial transpose** density matrix ρ^{PT}

$$\langle M'm' | \rho^{PT} | Mm \rangle = \langle Mm' | \rho | M'm \rangle \,. \tag{29}$$

- Applying this to the density matrix (18) we obtain

$$\rho^{PT} = \frac{1}{4} \begin{pmatrix} 1+P & 0 & 0 & 2P \\ 0 & 1-P & 0 & 0 \\ 0 & 0 & 1-P & 0 \\ 2P & 0 & 0 & 1+P \end{pmatrix} \,. \tag{30}$$

- A given density matrix ρ describes a separable state if **all** eigenvalues of ρ^{PT} are **positive**.
- In contrast, ρ describes an entangled system if **at least one** eigenvalue is **negative** (Horodecki *et al*, 1996; Peres, 1996).
- Calculating the **eigenvalues** λ_i of ρ^{PT} yields

$$\lambda_{1,2,3} = \frac{1}{4}(1-P) \,, \quad \text{and} \quad \lambda_4 = \frac{1}{4}(1+3P) \,. \tag{31}$$

Peres-Horodecki Criterion

- Eq. (31) indicate that all eigenvalues are positive for P-values in the range

$$-\frac{1}{3} \leq P \leq \frac{1}{3}\,, \qquad \text{(32a)}$$

- ρ is **separable** in this region.

ρ describes an **entangled** system in the range

$$-1 \leq P < -\frac{1}{3}\,, \qquad \text{(32b)}$$

where λ_4 becomes negative.

- The system is **maximally entangled** for $P = -1$; see Fig. 3.

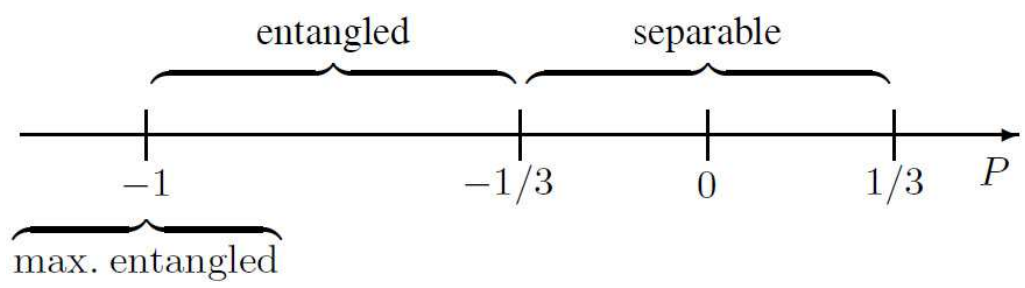

FIG. 3 Separable and entangled areas.

- Whether a given collision system is separable or non-separable depends on the **strength of the spin correlation** and can be decided experimentally by **measuring** $\mathbf{P}$.

Interrelation between spin-spin correlation and Bell's theorem (Bell, 1964).

Bell correlations

Bell's theorem

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✓ Entanglement is inherently a **non-local phenomenon**.

- In order to **avoid this *strange* feature** of quantum mechanics hypothetical **hidden variables** have been introduced (Einstein *et al*, 1935; Bell, 1964).
- Hoping to **reinstate** the results of **conventional quantum mechanics** in a local realistic way.
- Although designed to preserve locality it has turned out that attempts to do so failed in all circumstances.
- As a consequence of **Bell's theorem** (Bell, 1964) a set of inequalities have been derived, collectively known as **Bell inequalities**, which must be satisfied by any local theory, and which can be used as a **test of its validity**.

☞ A quantum mechanical state is said to display non-local correlations, or being **Bell (or EPR) correlated**, if it **violates** any of the **Bell inequalities**.

Bell's inequality

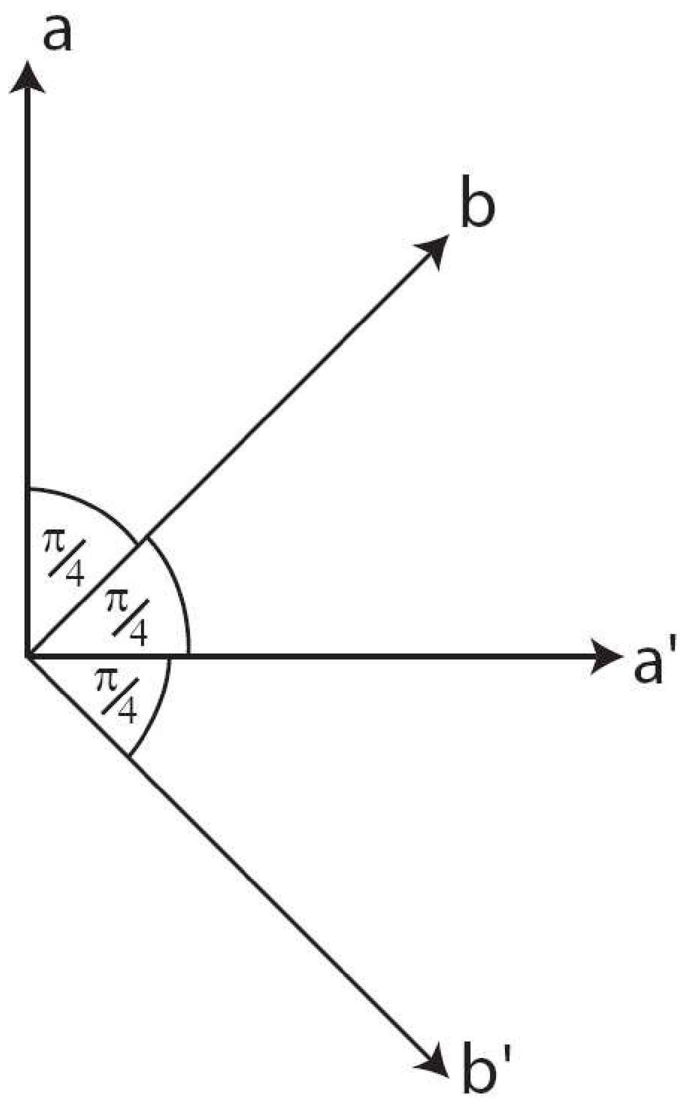

FIG. 4 Coplanar directions of the detector-spin-analyzer sets.

- Following Aspect *et al* (1982), we will adapt this to our case of interest.

☞ Consider the following **combination of correlation parameters**

$$\Delta(\mathbf{a},\mathbf{a}',\mathbf{b},\mathbf{b}') = \left|P_a^{(1)} \times P_b^{(2)} - P_a^{(1)} \times P_{b'}^{(2)}\right| + \left|P_{a'}^{(1)} \times P_b^{(2)} + P_{a'}^{(1)} \times P_{b'}^{(2)}\right| , \tag{33}$$

- where $\mathbf{a}, \mathbf{a}', \mathbf{b}$, and $\mathbf{b}'$ denote the **four directions** of the detector-spin-analyzer sets in the corresponding coincidence measurement of the first (directions $\mathbf{a}$ and $\mathbf{a}'$) and second beam (directions $\mathbf{b}$ and $\mathbf{b}'$), respectively.

Violation of CHSH inequality

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- ✓ It has been shown by Clauser *et al* (1969) that **local realism** implies the familiar **CHSH-inequality**

$$\Delta(\mathbf{a}, \mathbf{a}', \mathbf{b}, \mathbf{b}') \leq 2 \,, \tag{34}$$

- ✓ Quantum mechanics allows for a **larger bound** (Cirel'son, 1980)

$$\Delta(\mathbf{a}, \mathbf{a}', \mathbf{b}, \mathbf{b}') \leq 2\sqrt{2} \,. \tag{35}$$

- ■ In order to demonstrate that *local hidden variable theories* are **inconsistent with quantum mechanics** it is only necessary to show that condition (34) is **violated** for a particular setting.
- ☞ Consider **coplanar vectors**; Fig. 4.
- ■ Calculating the correlation parameters using (22) one obtains from (33)

$$☞ \quad \Delta = 2\sqrt{2}\,|P| \,. \quad ☜ \tag{36}$$

- 👍 Inequality (34) is **violated** for all correlation parameters with $P < -1/\sqrt{2}$.

☞ Thus, as depicted in Fig. 5, the spin-spin correlation system is entangled in the range $-\frac{1}{3} > P \geq -\frac{1}{\sqrt{2}}$ but **does not violate** any Bell inequalities.
The spin correlation in this region is not sufficiently strong.

- Thus, it is in principle possible to **simulate the quantum correlations** within local theories (**hidden variables**).

☞ This possibility is **ruled out** for values $-1 \leq P < -\frac{1}{\sqrt{2}}$ by Bell's theorem.

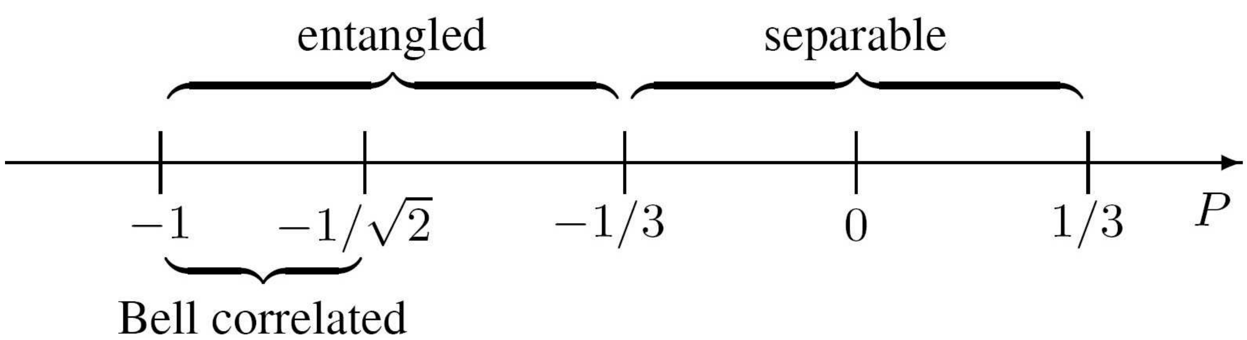

FIG. 5 Separable, entangled, and Bell correlated areas.

- In this case, the **correlations, generated by the spin-exchange collision, are stronger** than could ever be created by classical means and **cannot be described by local realistic theories**.

👍 For $P = -1$, (36) yields a **maximal violation of the CHSH inequality** (34), or any other of the Bell inequalities.

Experimental and Numerical Results

☞ $\mathbf{P}(\boldsymbol{\theta}, \mathbf{E}) = -\mathbf{A}_{\mathbf{ex}}(\boldsymbol{\theta}, \mathbf{E})$ ☜

Results – Hydrogen

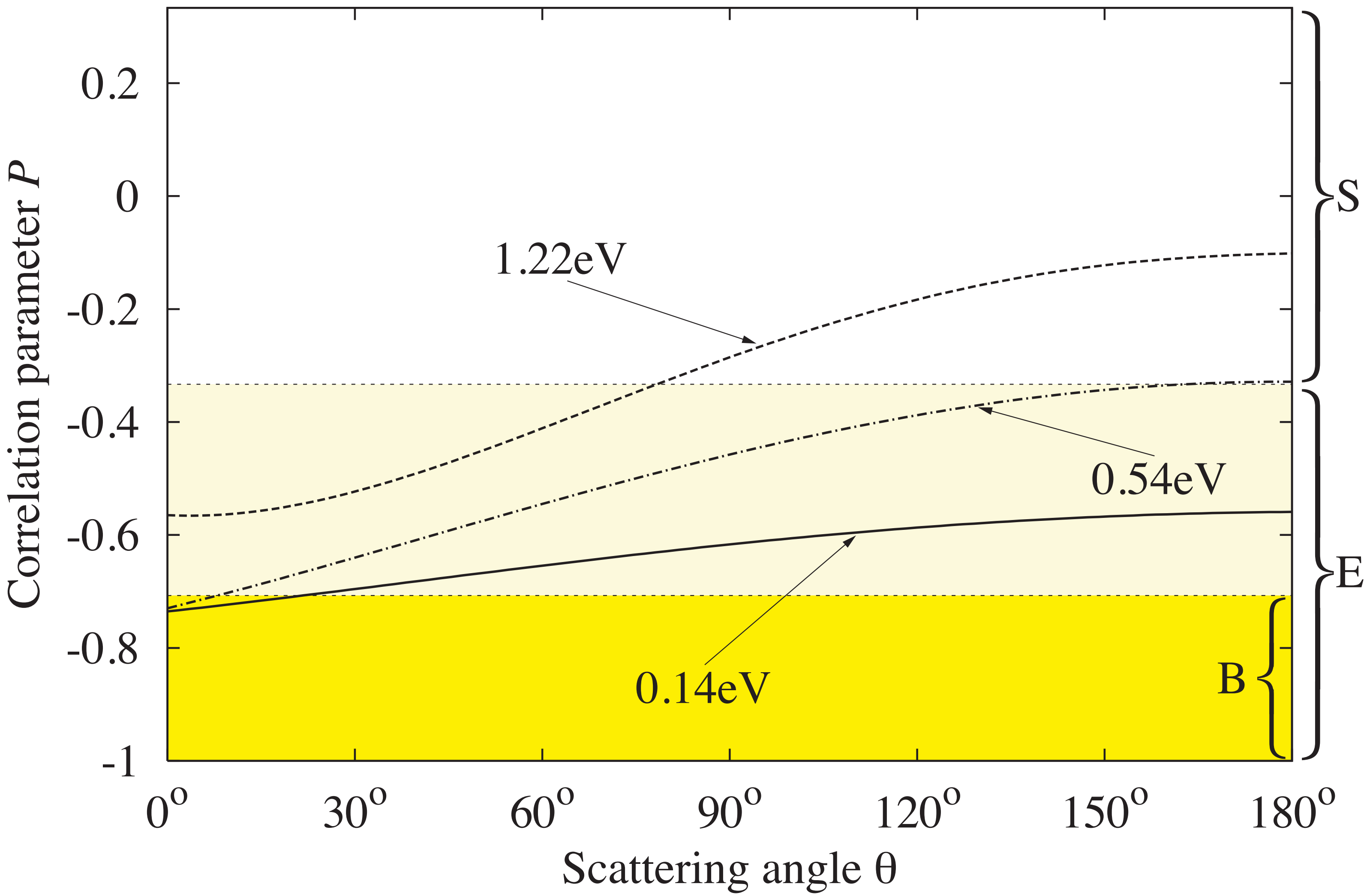

FIG. 6 Correlation parameter P of hydrogen versus scattering angle θ for different scattering energies. Num. data: 0.14 eV, 0.54 eV, and 1.22 eV, (MPCC) van Wyngaarden and Walters (1986). Horizontal lines divide separable (S), entangled (E), and Bell correlated (B) regions. The Bell correlated region can almost not be reached.

Results – Sodium I

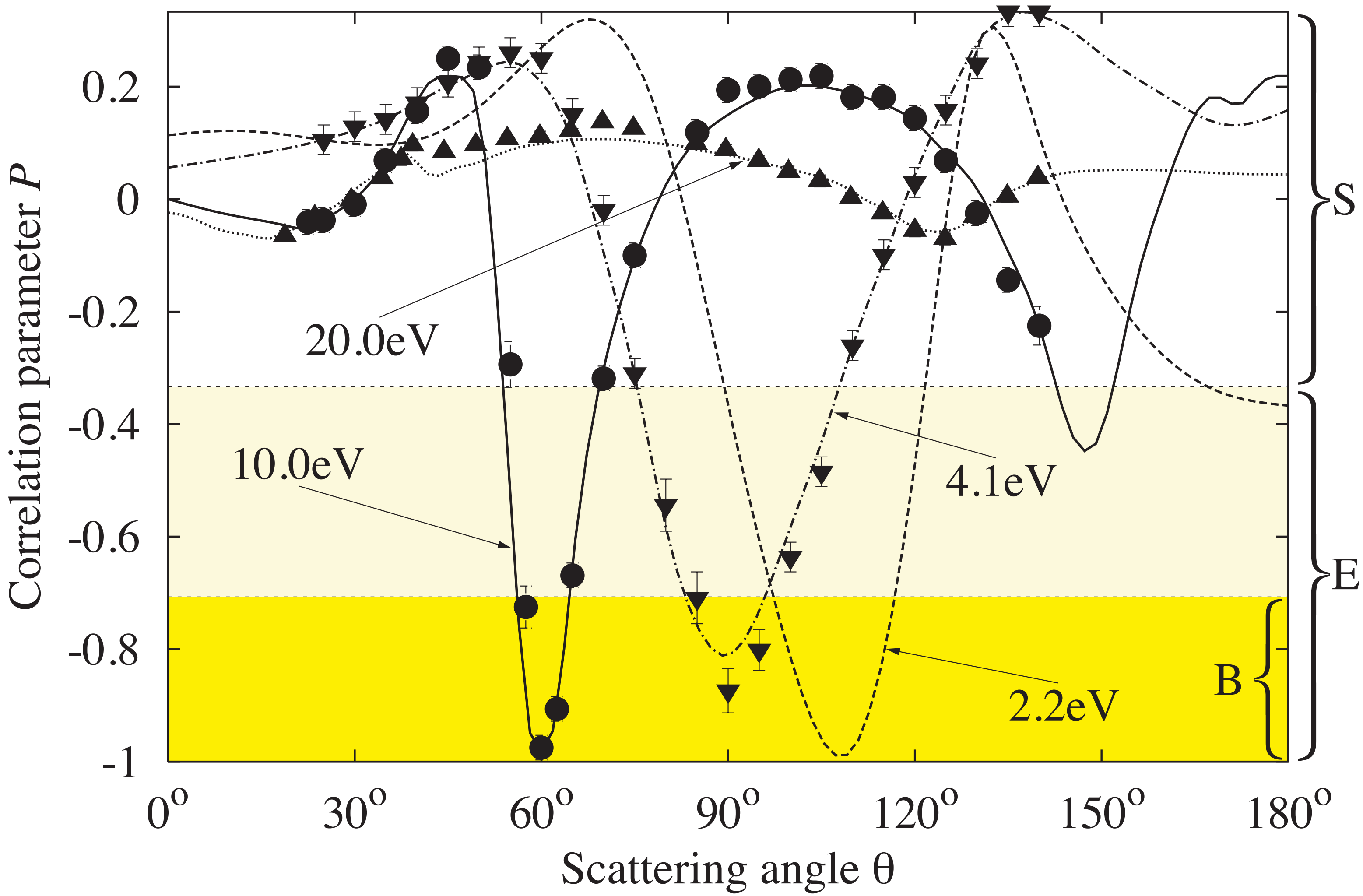

FIG. 7 Correlation parameter P of sodium versus scattering angle θ for different scattering energies. Exp. data by NIST group: 4.1 eV (▼), McClelland *et al* (1989, 1992); 10 eV (●), and 20 eV (▲), Kelley *et al* (1992). Num. data: 2.2 eV, (CC) Moores and Norcross (1972); 4.1 eV, 10 eV, and 20 eV (CCC) Bray (1994a,b). Horizontal lines divide separable (S), entangled (E), and Bell correlated (B) regions.

Results – Sodium II

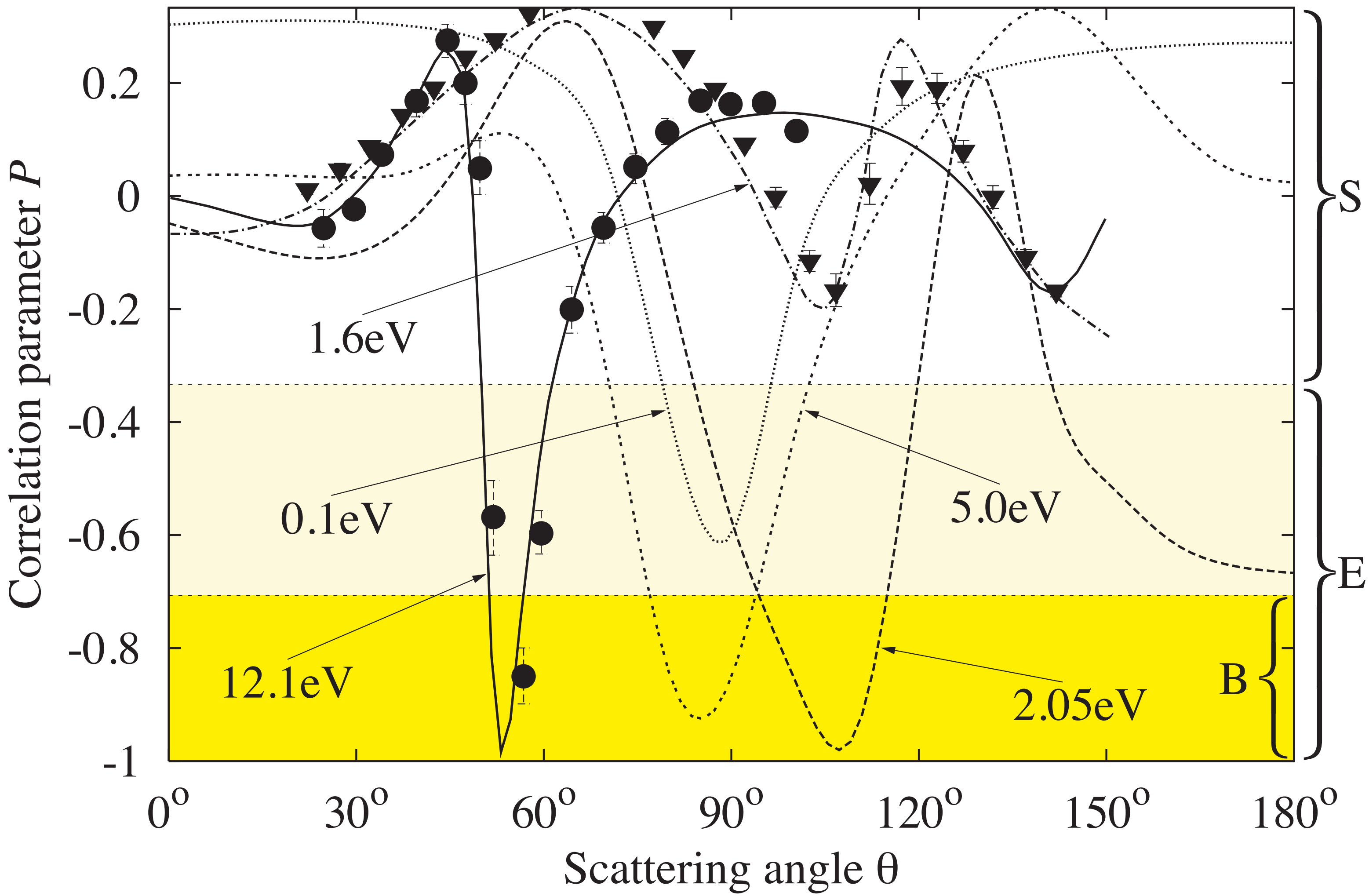

FIG. 8 Correlation parameter P of sodium versus scattering angle θ for different scattering energies. Exp. data by NIST group: 1.6 eV (▼), Lorentz *et al* (1991); 12.1 eV (●), McClelland *et al* (1992). Num. data: 0.1 eV, 2.05 eV, and 5.0 eV (CC) Moores and Norcross (1972); 1.6 eV, (CCC) Bray (1994b); 12.1 eV, (CCO) Bray and McCarthy (1993). Horizontal lines divide separable (S), entangled (E), and Bell correlated (B) regions.

Results – Lithium I

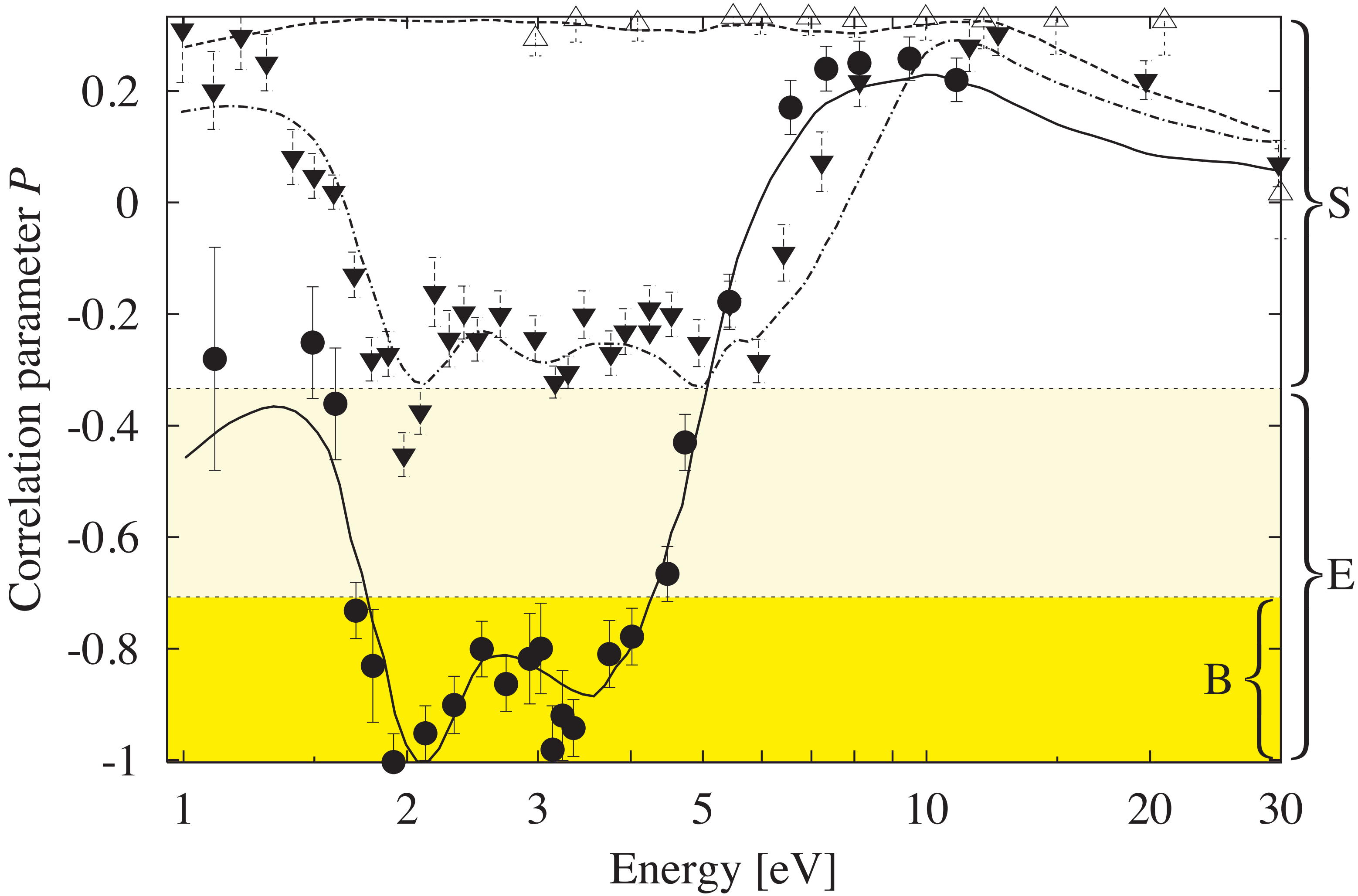

FIG. 9 Correlation parameter P of Li versus electron projectile energy in the range $E = 1 - 30\,\mathrm{eV}$ for fixed scattering angles $\theta = 65^o(\triangle), 90^o(\blacktriangledown), 107.5^o(\bullet)$. Exp. data: Baum *et al* (1986). Num. data: 13CCO8 calculation, Bray *et al* (1993). Horizontal lines divide separable (S), entangled (E), and Bell correlated (B) regions.

Results – Lithium II

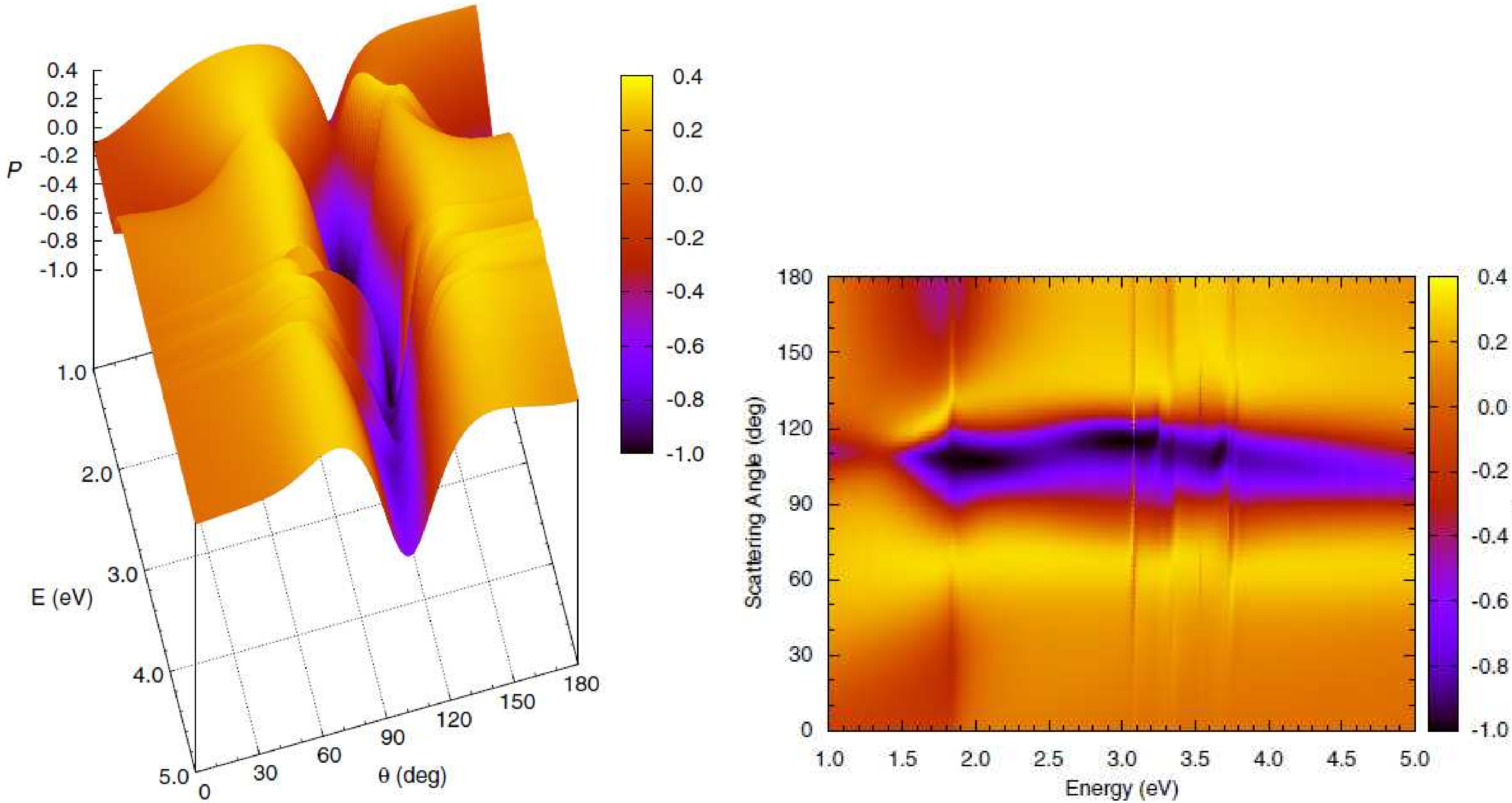

FIG. 10 Spin-correlation parameter P of Li versus electron projectile energy E and scattering angle θ as *(left)* 3D- and *(right)* contour plots, respectively. Num. data: 5-state close coupling R-matrix calculation, (Bartschat and Fonsecsa dos Santos, 2017).

Conclusions

Conclusions

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- **Analysis** of numerical and experimental **spin asymmetry** $\mathrm{A_{ex}}$ yields results for the **spin correlation parameter** P.
- **Li and Na data** reveal surprisingly **strong spin correlations**, created out of an initially **totally chaotic spin system**. – Unexpected result. –
- **Experimentally**, the entangled spin pairs can be determined via **one local measurement of** P.
- In particular, **Bell correlated spin pairs** have been obtained.

☝ The **CHSH-equation is maximally violated, with** $\mathrm{P} \simeq -1$.

- This is **proof** that the discussed **electron exchange process** is intrinsically **non-local**.
- Considering the remarkable experiments with photons (Hensen *et al*, 2015; Giustina *et al*, 2015; Shalm *et al*, 2015), $\longrightarrow$ we **emphasize** that

☞ **our results** originate from a **completely unrelated field** of research and refer to experiments with **massive particles**.

✓ Therefore, our results yield **additional support** to the thesis that **nature behaves quantum mechanically and non-local**.

Conclusions

- By the scattering experiment **Bell pairs** can be produced **quickly and repeatable** in the full range between $-\frac{1}{\sqrt{2}} > P \geq -1$.
- Studying the Li and Na data (Figs. 6–10) one can **tune to a particular scattering angle and energy**.

☞ Creating pairs of collision partners with any **desired degree of spin entanglement** $1/3 \geq P \geq -1$.

- In particular, the **singlet state** ($\mathbf{P = -1}$) can readily be generated in the collision.
- These **pairs** can then be used for **further experiments** where pairs of particles with a **high degree of correlation** are required.

👍 This might be of interest for quantum **communication** or **teleportation** studies and might lead to a certain **entanglement** between **scattering physics** and **quantum information**.

- New data for potassium are on their way (R-matrix); Bartschat *et al* (2017).

☺

Quantification of Entanglement
Werner States and Negativity

Negative Correlations

- For negative values of $P = -|P|$, that is anticorrelated spins, we can write the scattering matrix ρ in form of a Werner state (Werner, 1989)

$$\rho = \frac{1-|P|}{4}\mathbb{1} + |P|\big|00\big\rangle\big\langle 00\big|;\, , \tag{37}$$

with the singlet state $$\big|00\big\rangle = \frac{1}{\sqrt{2}}\big(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\big)\;. \tag{38}$$

- The Werner state (37) represents a mixture of the completely uncorrelated state $\sim \mathbb{1}$ (with amount $1-|P|$) and the maximally entangled singlet state.
- The magnitude $|P|$ of the correlation parameter plays the role of a mixing parameter in the Werner state.
- In the range $-\frac{1}{3} > P \geq -1$ the density matrix ρ (37) represents an entangled spin state (Peres-Horodecki condition).
- Presence of entangled spin pairs $\Longleftrightarrow$ verified experimentally by one local coincidence measurement, the determination of P.

Measure of Entanglement

- The amount of entanglement can be quantified using the concept of negativity (Zyczkowski *et al*, 1998; Vidal and Werner, 2002). Directly related to the Peres–Horodecki criterion.

☞ Definition:

$$N(\rho) = -2\sum_i n_i \,, \tag{39}$$

n_i are the negative eigenvalues of the partial transpose density matrix ρ^{PT}.

- If all eigenvalues $> 0\,,\quad \rho$ is separable, $\quad\Longrightarrow N(\rho) = 0$.
- Thus, $N(\rho)$ *measures* the amount by which ρ^{PT} fails to be positive definite.
- $N(\rho) \quad \sim \quad |P|$ as a measure for the entanglement in the system ρ (Vidal and Werner, 2002).
- Only the eigenvalue λ_4 can become negative. Hence, for $P < -\frac{1}{3}$,

$$N(\rho) = -2\lambda_4 = \frac{1}{2}\big(3|P| - 1\big)\,, \tag{40}$$

$$N(\rho) = 1 \qquad \text{for maximal entanglement,} \quad P = -1\,, \tag{41}$$

$$N(\rho) = 0 \qquad \text{for zero entanglement,} \quad P = -\frac{1}{3}\,. \tag{42}$$

Negativity of Werner states

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- Using (40), we are able to rewrite the Werner state (37) in the form $(-\frac{1}{3} > P \geq -1)$

$$\rho = \big(1 - N\big)\rho_2 + N\big|00\big\rangle\big\langle 00\big| \; . \tag{43}$$

- ρ_2 denotes the clearly separable density matrix with anticorrelated spins

$$\rho_2 = \frac{1}{6}\sum_i \big|\uparrow_i\big\rangle\big\langle\uparrow_i\big| \times \big|\downarrow_i\big\rangle\big\langle\downarrow_i\big| + \big|\downarrow_i\big\rangle\big\langle\downarrow_i\big| \times \big|\uparrow_i\big\rangle\big\langle\uparrow_i\big| \qquad i = x, y, z \; . \tag{44}$$

☞ Thus, the negativity $N(\rho)$ *measures* the amount of entanglement, contained in the system ρ, after all *classical* correlation $\sim \rho_2$ has been separated off from the original *non-local* singlet contribution.

- Only this fraction is useful for further entanglement studies.

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